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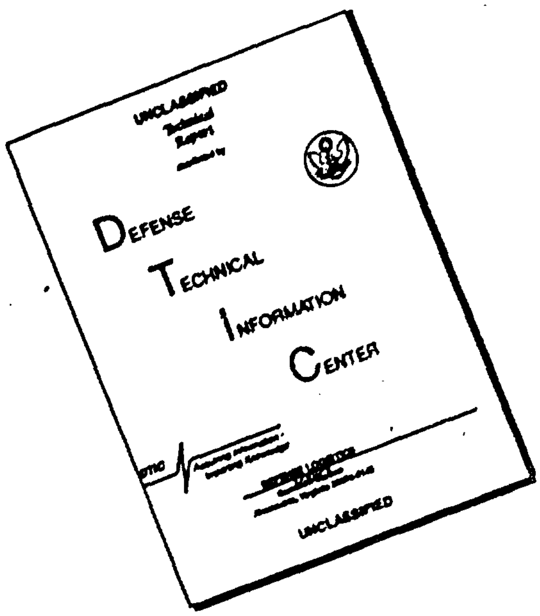

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ON SHIPS WITH ZERO AND SMALL WAVE RESISTANCE

By

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PREFACE

This technical report has been prepared specifically in connection with the Seminar on Theoretical Wave-Resistance to be held at the University of Michigan during August 1963. This report contains both new theoretical results, previously unreported, and material previously presented in HYDRONAUTICS Technical Report 117-2.

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NOTATION

a_n	Coefficients of polynomial representing source distribution
$A_1(\theta), A_2(\theta)$; $A(\theta), B(\theta)$	Wave amplitude functions
b_n	Coefficients of polynomial representing doublet distribution
B	Beam
f, f_1	Depths of end points of doublet distribution
F_H, F_L	Froude numbers with respect to draft and length respectively
g	Acceleration of gravity
H	Draft of ship
k_o	$= gH/V^2$
k_1	$= gL/V^2$
L	Length of ship
m(x)	Total strength of source per unit length
m(x,z)	Total strength of source per unit area
R	Ship wave resistance
$S_1(x), S_2(x)$	Functions of x defined by Equations [4] and [5]
V	Uniform velocity at ∞
x,y,z	Right handed rectangular coordinate system with z positive upward, x in the direction of the uniform flow velocity V, and origin on the mean free surface
x_1	Dummy variable indicating x coordinate of source distribution
z_1	Dummy variable indicating negative z coordinate of doublet distribution

ϵ_n	Perturbation coefficients of b_n
$\zeta_s, \zeta_{sB}, \zeta_{ss}$	Wave heights due to total ship, bow only, and stern only, respectively
ζ_B, ζ_{BF}	Wave heights due to infinite bulb and finite bulb respectively
ζ_{BL}	$= \zeta_B - \zeta_{BF}$
$- \mu$	Strength of doublet per unit length
λ	Strength of quadrupole

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INTRODUCTION

There are two ways of looking at the merits of a bulb on a ship. One is its effect on the interactions between the waves themselves and the other is its effect on the ship resistance itself as represented by these waves. Recently Inui, Takahei, and Kumano (1960) studied the bulbous bow ship from the former point of view by considering the bulb as a point doublet. They paid special attention to the 180° phase-difference between the bulb wave and the ship bow wave, and tried to make both amplitude functions the same in order to cancel out the wave. Wigley (1936) mentioned this wave phenomenon, but attacked the problem via the wave resistance approach. He derived six rules for the design of a bulb for a ship bow

This latter approach, involving the determination of the wave resistance directly, has been the conventional one used by most investigators. However, this generally leads to many cumbersome mathematical calculations. On the other hand, the method dealing with the interaction between the waves themselves can be simple and leads to a better physical insight. Using this approach, Takahei (1960) studied the "waveless bow" further and Kumano (1960) investigated the "waveless stern." They tried to match up the amplitude function of a doublet, which is a function of the doublet strength and the Froude number, as closely as possible with the amplitude function of a ship. However, it was not very clear how close the doublet distribution could be matched. It will be shown in this report that a doublet distribution whose amplitude function is exactly the same as that of the ship can be found but that it extends along a line from the free surface to infinite depth. With such a doublet distribution at the bow and stern the waves from the bow and stern of the given symmetric ship may be completely canceled and the ideal fluid wave resistance of the ship becomes zero. The semi infinite nature of the extension of the doublet distribution need not be a serious restriction since it will be shown that the effect of discarding that part of the doublet distribution extending from some finite depth to infinity can be made very small.

Further it is shown that a quadrupole distribution at the bow and the stern in addition to the doublet distribution offers practical advantages.

WAVES DUE TO A SHIP AND A BULB

As in the usual analysis, an inviscid, incompressible, and homogeneous fluid in steady flow with a free surface and of infinite depth is considered. The coordinate system O-xyz is right handed with origin on the mean free surface, x positive in the direction of the uniform flow velocity V, and z positive in the upward direction. In this report, except in the sections on two-dimensional streamlines the quantities x, y and other lengths except z are non-dimensionalized with respect to the ship length L; z is non-dimensionalized with respect to the draft H, and m is non-dimensionalized with respect to V and LH. All equations are expressed in non-dimensional form. All wave height components are assumed to be small and additive.

It is well known that a point source of strength m located at a point $(x_1, 0, -z_1)$, where $z_1 > 0$, produces a wave height non-dimensionalized with respect to L at large x given by (see Havelock, 1951),

$$\zeta_s = 8k_o \int_0^{\pi/2} m \quad \exp(-k_o z_1 \sec^2\theta) \sec^3\theta \cos\left[k_1(x-x_1)\sec\theta\right]$$

$$\times \cos(k_1 y \sin\theta \sec^2\theta)\, d\theta \qquad [1]$$

where

$$k_1 = \frac{Lg}{V^2} \text{ and } k_o = \frac{Hg}{V^2}.$$

Hence, for a line distribution of sources at y = 0, $z = -z_1$, $0 \le x_1 \le 1$ represented by the series,

$$m(x_1) = \sum_{n=0}^{\infty} a_n x_1^{\,n} \qquad [2]$$

the wave height will be

$$\zeta_s = 8k_o \int_0^{\pi/2} \int_0^1 m(x_1) \exp(-k_o z_1 \sec^2\theta) \sec^3\theta \cos\{k_1(x-x_1)\sec\theta\}$$

$$X \cos(k_1 y \sin\theta \sec^2\theta)\, dx_1\, d\theta$$

$$= \frac{8k_o}{k_1^{\,3}} \int_0^{\pi/2} \exp(-k_o z_1 \sec^2\theta) \Big[\sin(k_1 x \sec\theta) S_1(0)$$

$$- \cos(k_1 x \sec\theta) S_2(0) - \sin\{k_1(x-1)\sec\theta\} S_1(1)$$

$$+ \cos\{k_1(x-1)\sec\theta\} S_2(1) \Big] \cos(k_1 y \sin\theta \sec^2\theta)\, d\theta \qquad [3]$$

with

$$S_1(a) = \sum_{n=0}^{\infty} \frac{(-1)^n m^{(2n)}(a)}{(k_1 \sec\theta)^{2(n-1)}} \qquad [4]$$

$$S_2(a) = \sum_{n=0}^{\infty} \frac{(-1)^n m^{(2n+1)}(a)}{(k_1 \sec\theta)^{2n-1}} \qquad [5]$$

where

$$m^{(n)}(a) = \left(\frac{\partial^n m(x_1)}{\partial x_1^{\ n}}\right)_{x_1 = a} .$$

If we put

$$\zeta_s = \zeta_{sB} + \zeta_{ss} ,$$

where

$$\zeta_{sB} = \frac{8k_o}{k_1^{\ 3}} \int_0^{\pi/2} \exp(-k_o z_1 \sec^2\theta)\Big[\sin(k_1 \ x \sec \theta)S_1(0)$$

$$- \cos(k_1 \ x \sec \theta)S_2(0)\Big] \cos(k_1 y \sin \theta \sec^2\theta) \, d\theta, \qquad [6]$$

$$\zeta_{ss} = \frac{8k_o}{k_1^{\ 3}} \int_0^{\pi/2} \exp(-k_o z_1 \sec^2\theta) \Big[-\sin \{k_1 (x-1)\sec \theta\} \ S_1(1)$$

$$+ \cos \{k_1 (x-1) \sec \theta\}S_2(1) \cos (k_1 y \sin \theta \sec^2\theta\Big] \, d\theta,$$

[7]

then ζ_{sB} can be interpreted as the wave from the bow and ζ_{ss} as the wave from the stern according to the idea of the elementary wave (see Havelock, 1934a). We can see that both ζ_{sB} and ζ_{ss} are, in general, composed of sine and cosine elementary waves. Henceforth we will omit the word "elementary" except to avoid ambiguities. In the integrand of ζ_{sB} and ζ_{ss}, the coefficient of $\begin{smallmatrix}\sin\\ \cos\end{smallmatrix}$ $(k_1 x \sec\theta) \cos(k_1 y \sin\theta \sec^2\theta)$ is called an amplitude function.

For a ship of a finite draft, ζ_s has only to be integrated from $z_1 = 0$ to $z_1 = 1$. In this case $S_1(a)$ may be a function of z_1.

We now consider a doublet of strength $-\mu$ nondimensionalized with respect to V and LH^2 located at $x = 0$, $y = 0$, $z = -z_1$. The nondimensional wave height due to this doublet, at large x, is (see Wigley, 1936)

$$\zeta_B = -8k_o^2 \int_0^{\pi/2} \mu \ \exp(-k_o z_1 \sec^2\theta) \sec^4\theta \sin(k_1 x \sec\theta) \times \cos(k_1 y \sin\theta \sec^2\theta)\, d\theta \qquad [8]$$

where ζ_B is nondimensionalized with respect to L. Note that the sine waves originating from the origin have a negative sign in this case.

CONDITION OF NO WAVE RESISTANCE

We have seen from the previous section that the wave height due to a submerged body or a surface ship may be represented at large x by

$$\zeta = \int_0^{\pi/2} \left[A_1(\theta) \sin (k_1 x \sec \theta) + A_2(\theta) \cos (k_1 x \sec \theta) \right]$$

$$X \cos (k_1 y \sin \theta \sec^2 \theta) \, d\theta \qquad [9]$$

Havelock (1934b)showed that the nondimensionalized wave resistance represented by this wave system is

$$R = 32 \int_0^{\pi/2} \left(A_1^{\,2}(\theta) + A_2^{\,2}(\theta) \right) \cos^3\theta \, d\theta \qquad [10]$$

where R is related to the wave resistance R_o by $R = \dfrac{R_o}{\frac{\pi}{2}\rho L^2 V^2}$.

The integrand of this wave resistance integral is always positive. Hence, if there exists a ship whose wave resistance is zero, the necessary and sufficient condition for the no wave drag ship is

$$A_1(\theta) \equiv A_2(\theta) \equiv 0, \qquad [11]$$

(i.e. no wave itself). Of course the trivial solution for this is the case of no singularity or of an infinitely thin frictionless flat plate parallel to the uniform stream, i. e. $\mu = 0$ and $m = 0$.

However, there may be a non-trivial solution which makes $A_1(\theta) \equiv A_2(\theta) \equiv 0$ with the proper selection of the singularity distribution. Krein (1955) proved that there does not exist any finite ship whose Michell's resistance (the wave resistance represented by Michell's integral) becomes zero, but he cited a few examples of infinite convoys whose wave resistance is zero. In the next section we will find waveless solutions involving the appropriate combination of a doublet distribution with a source distribution. The basis for these is that while a positive sine wave usually starts from the bow of a ship without a bulb, a negative sine wave starts from the surface point directly above a bulb representable by a point doublet. This has been checked experimentally as well as theoretically by many investigators.

COMBINATION OF DOUBLET AND SOURCE DISTRIBUTION

The problem is therefore, how to find a doublet distribution for a given or otherwise determined proper source distribution in order to make the total wave amplitude function zero; i.e. find the doublet distribution $\mu(x_1, z_1)$ on the longitudinal center plane such that for a given source distribution $m(x_1, z_1)$ on the same plane

$$k_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(x_1, z_1) \quad \exp(-k_0 z_1 \sec^2\theta)\sec^3\theta \cos\left[k_1 (x-x_1)\sec\theta\right] dx_1 dz_1$$

$$-k_0^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu(x_1 z_1) \quad \exp(-k_0 z_1 \sec^2\theta)\sec^4\theta \sin\left[k_1 (x-x_1)\sec\theta\right] dx_1 dz_1 \equiv 0 \qquad [12]$$

Let us consider the simplest case first; vis., a uniform source distribution on the line $0 \leq x_1$, $y_1 = 0$, $z_1 = f$. To a first approximation this represents a semi-infinite paraboloid and is equivalent to the case of n = 0 in Equation [2]. Then the contribution to the wave from the bow is from Equation [6]

$$\zeta_{sB} = \frac{8k_o}{k_1} \int_0^{\pi/2} a_o \quad \exp(-k_o f_1 \sec^2\theta)\sec^2\theta \sin (k_1 x \sec \theta)$$

$$X \cos (k_1 y \sin \theta \sec^2\theta) \, d\theta \qquad [13]$$

Now this can be matched by a uniform doublet distribution b_o on a vertical line x = 0, y = 0, $f < z_1 < \infty$. Then the corresponding wave height will be, by an integration of [8] with respect to z_1 from f to ∞,

$$\zeta_B = - 8k_o \int_0^{\pi/2} b_o \quad \exp(-k_o f \sec^2\theta)\sec^2\theta \sin (k_1 x \sec \theta)$$

$$X \cos (k_1 y \sin \theta \sec^2\theta) \, d\theta \qquad [14]$$

From Equations [13] and [14] we now see that

$$\zeta = \zeta_{sB} + \zeta_B = 0$$

if
$$b_o = \frac{a_o}{k_1} .$$

Thus according to this result the waves from the point on the surface above the nose of a submerged semi-infinite paraboloid will be completely cancelled by a vertical circular cylinder extending from the nose of the paraboloid to infinite depth. This is the fundamental idea of how the wave amplitude function can be made equal to zero.

Let us now consider a little more general case of Equation [6]; e.g., an axially symmetric body. However, the source distribution of Equation [2], which is an ordinary power series, is not the proper one to use for the doublet distribution because this will produce cosine terms according to Equations [6] and [7] as well as sine terms, while our expression for the waves due to the doublet distribution at $x = 0$ gives only sine terms. Since there is no easy way to make the amplitude function of the cosine terms $A_2(\theta)$ of Equations [9] and [10] equal to zero by combination with a doublet distribution, one thing to do is to employ source distributions which do not give cosine terms. The other idea is to consider a different singularity distribution which produces cosine waves as well as sine waves to cancel ship waves This will be discussed later.

With reference to Equations [2], [8], [5], and [6], we note that $m^{(o)}(0) = a_o \quad m^{(1)}(0) = 2!a_2 ----m^{(2n)}(0) = (2n)!a_{2n}$ and

$$m^{(1)}(0) = a_1 .....m^{(2n + 1)}(0) = (2n + 1)!\ a_{2n + 1} \qquad [15]$$

i.e., $S_1(0)$ is related to only the even powers of the series [2] while $S_2(0)$ is related to only the odd powers of the series. The amplitude function of the sine terms in Equation [6] is

$$\frac{8k_0}{k_1^3} \quad \exp(-k_0 z \sec^2\theta)\ S_1(0),$$

and that of the cosine terms,

$$\frac{8k_0}{k_1^3} \quad \exp(-k_0 z \sec^2\theta)\ S_2(0).$$

We now see that a source distribution given by an even power series does not produce cosine terms. According to Weierstrass' approximation theorem, any continuous curve can be approximated by a polynomial in the domain $0 \leq x \leq 1$. If we consider another curve in $-1 \leq x \leq 0$, symmetric with respect to the first about $x = 0$ the polynomial which represents the whole curve in $-1 \leq x \leq 1$ must be an even power series. Hence we may say any curve in $0 \leq x \leq 1$ can be approximated by a polynomial of even powers in the domain. Equation [2] may therefore be written

$$m(x_1) = \sum_{n=0}^{\infty} a_{2n}\, x_1^{2n} . \qquad [16]$$

If we assume that our ship is symmetric about its midsection at $x = 0.5$ then $m(x_1)$ is odd with respect to this point or

$$m(0) = -m(1),\ m'(0) = +\ m'(1)$$

$$\text{-----} m^{(n)}(0) = (-1)^{n+1}\ m^{(n)}(1) \qquad [17]$$

Since from Equation [16] $m^{(2n+1)}(0) = 0$, from Equation [17] $m^{(2n+1)}(1) = 0$. Hence from Equation [5] $S_2(0) = S_2(1) = 0$.

Similarly from Equations [17] and [4] we can easily see

$$S_1(0) = -\ S_1(1)$$

Then from Equations [6] and [17], we see that the amplitude functions of ζ_{sB} and ζ_{ss} are exactly the same and both are sine waves.

When the body is not symmetric fore and aft with respect to the midsection, the wave from the stern can be treated separately in a similar manner by changing coordinates. If we take the origin at the stern, and express the source distribution for the body by an even power series, the wave system from the stern can be analyzed in exactly the same way as for the bow.

Therefore the solution to the zero wave drag problem for ζ_{ss} can be treated exactly in the same manner as that for ζ_{sB}.

We now consider only ζ_{sB} of Equation [6] corresponding to the source distribution of Equation [16] along $z_1 = f_1$, $y = 0$,

$$\zeta_{sB} = \frac{8k_o}{k_1^{\,3}} \int_0^{\pi/2} \exp(-k_o f_1 \sec^2\theta) \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!\, a_{2n}}{(k_1 \sec\theta)^{2(n-1)}} \sin(k_1 x \sec\theta)$$

$$\times \cos(k_1 y \sin\theta \sec^2\theta)\, d\theta \qquad [18]$$

The terms for large n's become very small for the usual range of Froude numbers, and in practice we have only to consider just a few terms of the series given by Equation [16] (see appendix). In order to find the corresponding doublet distribution which cancels the wave height ζ_{sB}, let $\mu(z_1)$, the doublet strength per unit length be expressed as

$$\mu(z_1) = \sum_{n=0}^{\infty} b_n (z_1 - f)^n \equiv \sum_{n=0}^{\infty} b_n \eta^n \qquad [19]$$

on the line $x = y = 0$, $f \le z_1 < \infty$. Thus for $f \le z_1 \le f_1$,

$$\zeta_{BF} = -8k_o^2 \int_o^{\pi/2} \int_o^{f_1 - f} \sum_{n=0}^{\infty} b_n \eta^n \sec^4\theta \quad \exp[-k_o(\eta+f)\sec^2\theta$$

$$X \sin(k_1 x \sec\theta) \cos(k_1 y \sin\theta \sec^2\theta)\, d\eta\, d\theta$$

$$= -8 \int_o^{\pi/2} \sum_{n=0}^{\infty} \frac{b_n n!}{k_o^{n-1} \sec^{2(n-1)}\theta} \left[\exp(-k_o f \sec^2\theta) - \exp(-k_o f_1 \sec^2\theta) \right.$$

$$\left. X \sum_{m=0}^{n} \frac{k_o^m \sec^{2m}\theta (f_1 - f)^m}{m!} \right] \sin(k_1 x \sec\theta) \cos(k_1 y \sin\theta \sec^2\theta) d\theta$$

[20,a]

The value of the second term for $f_1 \to \infty$ vanishes (see Appendix). Thus, for $f_1 \to \infty$,

$$\zeta_B = -8 \int_o^{\pi/2} \exp(-k_o f \sec^2\theta) \sum_{n=0}^{\infty} \frac{n! b_n}{k_o^{n-1} \sec^{2(n-1)}\theta} \sin(k_1 x \sec\theta)$$

$$X \cos(k_1 y \sin\theta \sec^2\theta)\, d\theta$$

[20,b]

If we compare Equations [18] and [20,b] we obtain

$$b_n = (-1)^n \frac{(2n)!}{n!} \frac{k_o^n}{k_1^{2n+1}} a_{2n} \qquad [21]$$

as the condition that the regular waves given by Equation [18] be exactly cancelled out by those given by Equation [20,b]. The doublet distributions [19] located at x = 0 and extending from $z_1 = f$ to $z_1 = \infty$ generate a wave system which for an ideal fluid cancells completely the bow wave system due to any given source distribution [Equation 16]. In a similar manner as mentioned before, the stern wave system can be completely eliminated

For the surface ship, we have only to consider, $m(x_1)$ of Equation [16] as the source strength per unit area on the center plane y = 0, for $0 \leq z_1 \leq 1$, $0 \leq x \leq 1$. If we integrate both [18] and [20] with respect to f from f = 0 to f = 1, we obtain the corresponding amplitude functions

$$A(\theta) \equiv \frac{8k_o}{k_1^3} \int_0^1 \exp(-k_o f \sec^2\theta) \sum_{n=0}^{\infty} \frac{(-1)^n (2n)! a_{2n}}{(k_1 \sec \theta)^{2(n-1)}} df, \qquad [22,a]$$

and

$$B(\theta) \equiv 8 \int_0^1 \exp(-k_o f \sec^2\theta) \sum_{n=0}^{\infty} \frac{n! b_n}{k_o^{n-1} \sec^{2(n-1)}\theta} df \qquad [22,b]$$

If Equation [21] holds for all depths f we then get

$$A(\theta) - B(\theta) \equiv 0.$$

Thus for any vertical distribution of sources, Equations [19] and [21] will give the required bow and stern bulb shape to completely cancel the ship waves.

It should be emphasized that the b_n in Equation [22] is no longer exactly the same as that obtained in the expansion of the doublet $\mu(z_1)$ in Equation [19]. From Equations [22], [19] and [20] the doublet strength per unit depth is now

$$\left.\begin{aligned} &\mu(z_1) = \int_0^{z_1} \sum_{n=0}^{\infty} b_n (z_1 - \zeta)^n \, d\zeta \\ &\text{in the domain } 0 \le z_1 \le 1, \text{ and} \\ &\mu(z_1) = \int_0^{1} \sum_{n=0}^{\infty} b_n (z_1 - \zeta)^n \, d\zeta \text{ for } z_1 \ge 1. \end{aligned}\right\} \quad [23,a]$$

For the case in which the source strength does not vary with depth

$$b_n = \text{const}$$

and

$$\frac{d\mu(z_1)}{dz_1} = \sum_{n=0}^{\infty} b_n z_1^{\,n}$$

or

$$\left.\begin{aligned} \mu(z_1) &= \sum_{n=0}^{\infty} \frac{b_n z_1^{\,n+1}}{n+1} \quad \text{for } 0 \le z_1 \le 1 \\ \text{and} \qquad \mu(z_1) &= \sum_{n=0}^{\infty} \frac{b_n}{n+1}\left[z_1^{\,n+1} - (z_1 - 1)^{n+1}\right] \quad \text{for } z_1 \ge 1 \end{aligned}\right\}[23,b]$$

Equations [23,b] show that the slope of the doublet distribution along the line $x = 0$, $y = 0$ has a discontinuity at $z_1 = 1$. Especially if we consider the one term $n = 0$ (which is an important term for the practical range of Froude numbers [see Appendix]) the bulb shape may be similar to a round pointed pencil.

In summary, we have shown how $A_1(\theta)$ and $A_2(\theta)$ of Equation [9] can be made identically zero. Thus a means of satisfying the necessary and sufficient condition for a zero wave drag ship in an ideal fluid has been obtained. Unfortunately this requires the use of bulbs of infinite draft. In a later section we will consider the effect of cutting off the doublet distribution (or bulb) at some finite depth.

QUADRUPOLE DISTRIBUTIONS

We have represented the ship shape in the preceding by an infinite even power series (16). However, the series may not converge too rapidly, and for the higher Froude numbers, which we may wish to consider, the higher order terms in the series (16), can not be neglected. Suppose we have an arbitrarily specified polynomial of finite terms which represents the source distribution on the center plane (see Weinblum, 1950). Then we have to consider the odd power terms in Equation [2] and thus the cosine wave system occurs as well as the sine wave system in Equation [6] and [7], starting at the bow and the stern. We therefore have to consider some means of cancelling cosine waves.

A point doublet is a one step higher order singularity than a point source; the wave or the flow field due to a doublet is therefore represented by a derivative of that due to a source with respect to the position of the source (same as the position of the doublet) in the direction of the doublet (see Milne-Thomson). It is well known that a submerged point source produces positive cosine waves (see Equation [1]). Therefore, the negative sine wave can be produced by a negative doublet which produces a closed body in the uniform stream.

Now we consider a one step higher order singularity than a doublet which is called a quadrupole. That is, we consider two doublets of opposite signs, with the magnitude of each strength μ, and the distance between them, a, nondimensionalized with respect to H. Along the same idea of forming a doublet by a source and a sink (see Milne-Thomson 1955), we form a quadrupole by making

$$\lim_{a \to 0} a\mu = \text{finite constant, say, } \lambda \qquad [24]$$

which is the strength of the quadrupole. Its waves will be the derivative of the wave height due to a point doublet with respect to the position of the doublet in the direction of the quadrupole, namely cosine waves. The sign of the waves will depend on the sign (direction) of the quadrupole. That is, a point quadrupole with the strength λ_o at $x = 0$, $y = 0$, $z = -z_1$ in the uniform flow V generates the wave height

$$\zeta_q = - 8k_o^3 \int_0^{\pi/2} \lambda \exp(-k_o z_1 \sec^2\theta)\sec^5\theta \cos(k_1 x \sec \theta)$$

$$X \cos (k_1 y \sin \theta \sec^2\theta) \, d\theta \qquad [25]$$

where $\lambda = \lambda_o/(H^3 L \quad)$

There are two great features of the quadrupole which will be mentioned in the following sections. One is the same feature which the doublet distribution had with respect to the even power terms of the ship source distribution, but in this case with respect to the odd terms. The other is the composition of a point singularity which produces no wave - say, a no-wave-singularity.

COMBINATION OF QUADRUPOLE AND SOURCE DISTRIBUTIONS

We have mentioned that the odd power terms of the ship source distribution [2] produce cosine waves and that the quadrupole also produces cosine waves. Otherwise, both amplitude functions have similar qualities to those of the even power terms of Expression [2] and for the doublet. Namely, if we consider the odd power terms of Expression [2] say

$$m = a_{2n+1} x_1^{2n+1}, \quad \text{in} \quad \begin{array}{l} 0 \le x_1 < 1, \\ 0 \le z_1 \le f, \end{array} \qquad [26]$$

then the wave due to this source distribution starting from the bow x = 0 is from Equation [6]

$$\zeta = -8 \int_0^{\pi/2} a_{2n+1} [1-\exp(-k_o \sec^2\theta)](-1)^n \, (2n+1)! \quad X$$

$$X \; k_1^{-2(n+1)} \cos^{2n+1}\theta \cos(k_1 x \sec\theta) \cos(k_1 y \sin\theta \sec^2\theta) d\theta$$

[27]

In exactly similar manner as in matching the doublet distribution to the even power terms of the ship source distribution in order to cancel the bow waves, the corresponding quadrupole distribution may be put as in Equation [23,b]

$$\lambda = b_{n+1} \frac{z_1^{\,n+2}}{n+2} \qquad \text{in } 0 \le z_1 \le 1 \qquad [28]$$

$$\lambda = b_{n+1} \frac{[z_1^{\,n+2} - (z_1-1)^{n+2}]}{n+2} \qquad \text{in } 1 \le z_1 \le \infty \qquad [29]$$

then from Equation [25], the wave due to this quadrupole becomes

$$\zeta_q = -\,8k_1 \int_0^{\pi/2} b_{n+1} \quad 1\text{-}\exp(-k_o \sec^2\theta) \quad (n+1)!\ k_o^{\,n-1} \quad X$$

$$X \cos^{2n+1}\theta \ \cos(k_1 x \sec\theta)\ \cos(k_1 y \sin\theta \sec^2\theta)d\theta \qquad [30]$$

Hence, if we make

$$b_{n+1} = \frac{(-1)^{n+1} k_o^{\,n+1}(2n+1)!}{(n+1)!\,k_1^{\,2n+3}} \ a_{2n+1} \qquad [31]$$

the wave due to the odd power term of the source distribution will be completely cancelled. The above argument holds for any integer n. The stern waves due to the terms of odd power in the source distribution can be dealt with exactly in the same manner as those of the bow.

In general, the bow waves have stronger sine components than cosine components, the former depending mostly on the entrance angle of the ship bow. Therefore, the doublet distribution is more important than that of the quadrupole. In fact, this is the reason, which will be shown later, that the combination in practice of optimum distributions of doublets and quadrupoles can produce a closed body, although the quadrupole itself can not.

NO-WAVE-SINGULARITY

By Krein's proof (1955) we know that there exists no singularity distribution which produces a finite closed body and yet produces no wave. However, it would be very useful if we knew of some singularity which by itself may not produce a closed body, but may deform some closed body produced by another singularity distribution when both are combined together, and which in combining produces no additional wave resistance. Krein (1955) first found such a special function h(x,z) which consists of an arbitrary nice function $\Phi(x,z)$ of x,z,

$$\frac{\partial^2 \Phi}{\partial x^2} + k_o \frac{\partial \Phi}{\partial z} \equiv h(x,z) \qquad [32]$$

with boundary conditions

$$\Phi = \frac{\partial \Phi}{\partial x} = 0 \qquad [34]$$

on the boundary of the domain of the singularity distribution on the center plane of a ship. He proved that the Michell's wave resistance due to a ship surface $f(x,z)$ is exactly the same as that due to a ship surface $f(x,z) + h(x,z)$. That is, there are infinitely many ship body forms which have the same resistance and volume.

Here we will see that if we use a point quadrupole in the x direction and a point doublet in the z direction at the same point, the combined singularity will not produce any wave in far downstream. The wave height due to a doublet in the z direction at the point $(0,0,z_1)$ is

$$\zeta_d = 8k_o^2 \int_0^{\pi/2} \mu \exp(-k_o z_1 \sec^2\theta)\sec^5\theta \cos(k_1 x \sec\theta) \times$$

$$\times \cos(k_1 y \sin\theta \sec^2\theta)\, d\theta \qquad [35]$$

Hence if we compare Equation [25] with Equation [35] it will be easily seen that

$$\zeta_d + \zeta_q = 0 \qquad [36]$$

for

$$k_o\lambda = \mu \qquad [37]$$

Since this is a point singularity and it does not produce any wave, it can be distributed in an arbitrary manner anywhere on top of the ship singularity distribution without changing the original ship wave resistance.

In fact, it can be shown that if this no-wave-singularity is distributed continuously, this will be in effect the same as Krein's form. Besides, by differentiating both wave heights ζ_d and ζ_q by the same parameter we may be able to construct innumerable higher order no-wave-singularities. Therefore in this report we will call the aforesaid no-wave-singularity as that of the first order.

BODY STREAMLINE SHAPE DUE TO COMBINATION OF SINGULARITIES IN THE UNIFORM FLOW

Now to utilize the quadrupole in practice we have to investigate the way in which it produces bodies when combined with other singularities in the uniform stream.

To be more general, we may further have to consider the discontinuity (shoulder) in the tangent of the waterlines. Then not only the negative doublet but also the positive doublet at the slope discontinuity may be needed to cancel the negative sine waves.

In the following subsections we will consider body shapes which may be produced by combinations of various singularities which may be utilized in improving ship shapes in order to decrease their wave resistance.

The streamlines produced by the three dimensional singularities in the uniform flow may be obtained by solving the streamline equations

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \qquad [38]$$

where

$$u = -\frac{\partial\phi}{\partial x} + V, \quad v = -\frac{\partial\phi}{\partial y}, \quad w = -\frac{\partial\phi}{\partial z} \qquad [39]$$

represented by distributions of singularities. This could be solved by Runge-Kutta-Gills (Gill, 1951) numerical method. However, the exact form of ϕ due to the singularity under the free surface even under the assumption of the linear boundary conditions is so complicated, that it becomes practically very advantageous to relate the exact shape of the ship to the source distribution, on the assumption of zero Froude number. For example, Inui (1957) used this approximation (double model); he verified the sufficient validity of his method for moderate Froude numbers by comparing theory and his experiments. This method could be used here to find the relation between singularity distributions and ship shapes, as in the case of ships with no-wave singularities and with bulbs made of doublets and quadrupoles. However, considerable insight is gained with much greater simplicity by considering only special simple cases, as the two-dimensional case for instance. Herein we discuss the problem using dimensional quantities unless specified.

TWO-DIMENSIONAL DOUBLET AND QUADRUPOLE

The complex potential due to a doublet with the strength $-\mu < 0$ and a quadrupole with its strength $\lambda > 0$, with both directions parallel to the x axis, in a uniform flow is

$$w = \Phi + i\Psi = -\,Vz - \frac{\mu}{z} + \frac{\lambda}{z^2} \qquad [40]$$

where $$z = x + iy$$

Hence the stream function is

$$\Psi = -\,Vy + \frac{\mu y}{x^2+y^2} - \frac{2\lambda xy}{(x^2+y^2)^2}$$

$$= \frac{-y}{x^2+y^2}\left[V\,(x^2+y^2) - \mu + \frac{2\lambda x}{x^2+y^2}\right] \qquad [41]$$

If we use polar coordinates

$$x = r\cos\theta, \quad y = r\sin\theta$$

we can easily see that the dividing streamline is

$$y = 0$$

and

$$r^2 = \frac{\mu}{V} - 2\,\frac{\lambda}{V}\,\frac{\cos\theta}{r} \qquad [42]$$

This means that the body is deformed from the sphere

$$r^2 = \frac{\mu}{V} \qquad [43]$$

extending (or shrinking) r by an approximate amount of $\frac{2\lambda}{rV}\cos\theta$ in each direction. In order to have a closed body such that all streamlines inside the surface streamline are wholly contained, the magnitude of λ should not be too large. To obtain the limiting value of λ, we have only to consider the point $\theta = 0$. We put

$$f(r) = r^3 - r\frac{\mu}{V} + \frac{2\lambda}{V} \qquad [44]$$

and notice $f(r) = 2\frac{\lambda}{V} > 0$ when both $\gamma = 0$ and $\gamma = \sqrt{\frac{\mu}{V}}$. And

$$f'(r) = 0 \text{ at } r = \sqrt{\frac{\mu}{3V}}$$

Hence if

$$f\left(\sqrt{\frac{\mu}{3V}}\right) = -\sqrt{\frac{\mu}{3U}}\left(\frac{2\mu}{3V}\right) + \frac{2\lambda}{V} \leq 0$$

or

$$\frac{\lambda}{V} \leq \left(\frac{\mu}{3V}\right)^{\frac{3}{2}} \qquad [45]$$

there exist two roots of $f(r) = 0$ in $0 \leq r \leq \sqrt{\frac{\mu}{U}}$. That is, when λ becomes large from $\lambda = 0$ within the limit of the inequality (45), Equation [44] represents two separate closed streamlines such that one is inside the other (see Figure 5a). If λ becomes larger than $\left(\frac{\mu}{3V}\right)^{\frac{3}{2}}$, the inner streamlines which were inside the inner closed streamline when $\frac{\lambda}{V} < \left(\frac{\mu}{3V}\right)^{\frac{3}{2}}$, pop out and there will be only one closed streamline (see Figure 5b). Therefore λ should satisfy the inequality (45) in order that a meaningful body exist. Figures 6 and 7 show examples of the dividing streamlines. When λ is negative the body form will be reversed along the x axis, and the condition on the magnitude of λ and μ is exactly the same as in the previous case.

A SIMPLE SOURCE AND A DOUBLET IN A UNIFORM STREAM

Consider a point doublet with the strength $\mu > 0$ at $z = x + iy = 0$ combined with a point source with its strength $m > o$ at $z = z_o$. Then the complex potential w will be written

$$w = - Vz - m \log (z-z_o) + \frac{\mu}{z} \tag{46}$$

The stream function is

$$\Psi = - Vy - m \tan^{-1} \frac{y-y_o}{x-x_o} - \frac{\mu y}{x^2+ y^2} \tag{47}$$

By

$$x = r \cos \theta, \ y = \sin \theta$$

$$\Psi = - Vr \sin \theta - m\theta_1 - \frac{\mu \sin \theta}{r} \tag{48}$$

where

$$\theta_1 = \text{arc tan} \left(\frac{y-y_o}{x-x_o}\right)$$

Hence the body streamline is obtained by putting $\Psi = -m\pi$. Non-dimensionalizing Equation [48] by $Vh \equiv m\pi$

$$\bar{r}^2 \sin\theta + \bar{r}\,(\theta_1/\pi - 1) + \bar{\mu}\sin\theta = 0 \qquad [49]$$

The streamline due to a source in the uniform flow is well known (see e. g. Milne-Thomson, 1955 p. 199). The combination of source and a positive doublet produces a neck. The strength of the doublet should satisfy

$$\bar{\mu} \equiv \frac{\mu}{Vh^2} < \frac{1}{4} \qquad [50]$$

in order for the body streamline to make physical sense. The dividing streamline is plotted in Figure 8 as an example, for a special value of μ.

A SIMPLE SOURCE AND A NO-WAVE-SINGULARITY OF THE FIRST ORDER IN A UNIFORM STREAM

We consider the no-wave-singularity, a quadrupole in the x direction with strength λ and a point doublet in the z direction with the strength μ, both located at the origin, and a point source at z_o in two dimensions. The complex potential due to these singularities is

$$w = \Phi + i\Psi = -Vz + i\,\frac{\mu}{z} + \frac{\lambda}{z^2} - m\log(z - z_o) \qquad [51]$$

Hence the stream function is

$$\Psi = -Vy + \frac{\mu x}{x^2 + y^2} - \frac{2\lambda xy}{(x^2 + y^2)^2} - m \tan^{-1} \frac{y - y_o}{x - x_o} \qquad [52]$$

Since the no-wave-singularity does not produce any extra fluid, $\Psi = -m\pi$ which is the dividing streamline for the half body due to a simple source in the uniform stream will still be the dividing streamline in this case.

If we nondimensionalize Ψ by hV where $h = \frac{m\pi}{V}$ is the radius of the half body due to a point source, the dividing streamline can be written as

$$-1 = -\bar{y} - \frac{\theta_1}{\pi} + \frac{\bar{x}\,\bar{\mu}}{(\bar{x}^2 + \bar{y}^2)^2} \left[x^{-2} + \left(\bar{y} - \frac{\bar{\lambda}}{\bar{\mu}} \right)^2 - \left(\frac{\bar{\lambda}}{\bar{\mu}} \right)^2 \right] \qquad [53]$$

where

$$\bar{y} = y/h, \quad \bar{x} = x/h, \quad \bar{\mu} = \mu/(Vh^2), \quad \bar{\lambda} = \lambda/(Vh^3), \quad \theta_1 = \tan^{-1} \frac{y - y_o}{x - x_o}.$$

Although it is very hard to see the exact shape of the dividing streamline without plotting x and y of Equation [53] it may be easy to see just how the original half body streamline is distorted by the no wave singularity. The last term in Equation [53] is positive inside the circle K (center at $(0, \frac{\bar{\lambda}}{\bar{\mu}})$; radius, $\frac{\bar{\lambda}}{\bar{\mu}}$) and $\bar{x} < 0$, and outside the circle K and $\bar{x} > 0$; and negative inside the circle K and $\bar{x} > 0$, and outside K and $\bar{x} < 0$. When $\bar{x}^2 + \bar{y}^2 >> 1$ the distortion of the original dividing streamline is small. The distorted streamline will be of the shape shown in Figure 9. The no-wave-singularity has from Equation [37] the property of

$$\frac{\bar{\lambda}}{\bar{\mu}} \doteq \frac{V^2}{gh} \equiv \frac{1}{k_h} \equiv F_h^{\,2}$$

For $L < 6h$ and $F_L > 0.3$, $\bar{\lambda}/\bar{\mu} > 0.5$. Hence, for the moderate F_L, the circle K may cut the upper dividing streamline but not the lower one and there are two wiggles in the upper dividing streamline and only one wiggle at lower one as in Figure 9.

FINITE DOUBLET DISTRIBUTION

SUBMERGED AXISYMMETRIC BODY WITH FORE AND AFT SYMMETRY

In order to determine the effect of limiting the doublet distribution to a finite depth we examine the difference between the expressions for ζ_{BF} as given by Equation [20,a] with that for ζ_B of Equation [20,b].

$$\zeta_R = 8\int_0^{\pi/2} \exp(-k_o f_1 \sec^2\theta) \sum_{n=0}^{\infty} \frac{b_n n!}{k_o^{\,n-1} \sec^{2(n-1)}\theta} \sum_{m=0}^{n} \frac{k_o^m \sec^{2m}\theta\,(f_1 - f)^m}{m!}$$

$$\times \sin(k_1 x \sec\theta)\cos(k_1 y \sin\theta \sec^2\theta)\, d\theta \qquad [54,a]$$

This is the uncancelled wave system resulting from cutting off the doublet distribution at depth f_1. Thus instead of each term of the series in $\zeta_B + \zeta_{SB}$ being zero there will be a remainder,

$$\zeta_{Rn} = \int_0^{\pi/2} \exp(-k_o f_1 \sec^2\theta)\, \frac{b_n n!}{k_o^{n-1} \sec^{2(n-1)}\theta} \sum_{m=0}^{n} \frac{k_o^m \sec^{2m}\theta (f_1 - f)^m}{m!}$$

$$\times \sin(k_1 x \sec\theta)\cos(k_1 y \sin\theta \sec^2\theta)\, d\theta \qquad [54,b]$$

The magnitude of ζ_{Rn} can be easily estimated as

$$\left.\begin{aligned} \zeta_{Rn} &< 8 \quad \exp(-k_o f_1)\, \frac{b_n n!}{k_o^{n-1}} \sum_{m=0}^{n} \frac{k_o^m (f_1 - f)^m}{m!} \\ \text{e.g.} \quad \left|\zeta_{Ro}\right| &< 8 \quad \exp(-k_o f_1) \\ \left|\zeta_{R1}\right| &< 8 \quad \exp(-k_o f_1\, b_1 \left\{1 + k_o(f_1 - f)\right\} \text{ etc.} \end{aligned}\right\} [54,c]$$

For the usual range of Froude numbers, ζ_{Ro} is the dominant term (see Appendix). This decreases exponentially with increasing $k_o f_1$. Although for increasing n, ζ_{Rn} decreases more slowly with increasing $k_o f_1$, its magnitude becomes increasingly smaller than ζ_{Ro} . Therefore, even if the wave amplitude is not completely cancelled out the doublet distribution still has a substantially favorable effect on each term.

SHIP WITH FORE AND AFT SYMMETRY

For a finite draft ship whose source distribution is independent of z, Equations [20], [21] and [22,b] give for the wave height at large x due to the infinitely deep doublet distribution alone

$$\zeta_B = -8\int_0^{\pi/2} (1- \exp(-k_o \sec^2\theta) \sum_{n=0}^{\infty} \frac{n!\, b_n}{k_o{}^n \sec^{2n}\theta} \sin (k_1 x \sec \theta)$$

$$\times \cos (k_1 y \sin \theta \sec^2\theta)\, d\theta \qquad [56,a]$$

Equation [56,a], of course, also represents the negative of the wave system due to the ship bow where b_n is obtained from Equation [21].

If we omit the doublet distribution from the point $z_1 = f_1 > 1$ to $z_1 = \infty$, we obtain the uncancelled wave height due to this by integrating the second term in Equation [20,a] with respect to f from 0 to 1.

$$\zeta_R = \sum_{n=0}^{\infty} \zeta_{Rn} = 8\int_0^{\pi/2} \exp(-k_o f_1 \sec^2\theta) \sum_{n=0}^{\infty} b_n n! \sum_{r=0}^{n} \frac{\{f_1{}^{r+1} - (f_1 - 1)^{r+1}\} k_o{}^{r-n+1}}{(r+1)\,!\, \sec^{2\,(n-r-1)}\theta}$$

$$\times \sin (k_1 x \sec \theta) \cos(k_1 y \sin \theta \sec^2\theta)\, d\theta \qquad [56,b]$$

This is formally similar to Equation [24,a]. As in Equations [24,b] and [24,c], we can write

$$\left.\begin{aligned} \zeta_{Ro} &< 8 \quad \exp(-k_o f_1) \quad b_o k_o \\ \zeta_{R_1} &< 8 \quad \exp(-k_o f_1) \quad b_1 \left(1 - \frac{1-2f_1}{2}\right) k_o \\ &\text{etc.} \end{aligned}\right\} \qquad [56,c]$$

Therefore the same comment about Equation [54,a] can be applied to Equation [56,a] that is, the uncancelled wave height, ζ_R of Equation [56,b] decreases almost exponentially with increasing $k_o f_1$ for the usual range of Froude numbers.

To obtain the approximate correction we may use a perturbation method. We replace b_n by $b_n(1+\epsilon_n)$ in Equations [56,a] and [56,b]. The sum of these two newly formed equations gives the wave height produced by the perturbed doublet distribution of depth f. When we subtract from this the wave system due to the ship bow as given by the negative of Equation [56,a] we obtain for each term of the resulting series

$$I_n = -8 \, b_n \int_0^{\pi/2} \left\{ \frac{(1- \exp(-k_o \sec^2\theta))}{k_o^{\,n} \sec^{2n}\theta} \epsilon_n - (1+\epsilon_n) \frac{\exp(-k_o f_1 \sec^2\theta)}{k_o^{\,n} \sec^{2n}\theta} \right.$$

$$\left. \times \sum_{r=0}^{n} \frac{\left[f_1^{\,r+1} - (f_1 - 1)^{r+1}\right]}{(r+1)!\cos^{2(r+1)}\theta} k_o^{\,r+1} \right\} \sin(k_1 x \sec\theta)\cos(k_1 y \sin\theta \sec^2\theta) d\theta. \qquad [57,a]$$

The wave resistance represented by the first term in the wave system I_o (putting n = 0 in Equation [57,a] except for a constant factor is

$$R(I_o)=\int_0^{\pi/2}\left\{\epsilon_o\left(1-e^{-k_o\sec^2\theta}\right)-(1+\epsilon_o)e^{-k_o f_1\sec^2\theta}k_o\sec^2\theta\right\}^2\cos^3\theta d\theta\ d\theta$$

$$=\epsilon_o^2\left\{\frac{2}{3}+2\,e^{-k_o}G_{-2}(k_o)-4e^{-k_o/2}G_{-2}\left(\frac{k_o}{2}\right)\right\}-2\epsilon_o(1+\epsilon_o)\,k_o$$

$$\times\left\{e^{-k_of_1/2}G_{-1}\left(\frac{k_of_1}{2}\right)-e^{-(k_o+k_of_1)/2}G_{-1}\left(\frac{k_o+k_of_1}{2}\right)\right\}$$

$$+\ k_o^2\ (1+\epsilon_o)^2\ \frac{e^{-k_of_1}}{2}\ K_o\ (k_of_1)$$

where

$$\int_0^{\pi/2}\exp(-2k_o\sec^2\theta)\sec^{2n+1}\theta d\theta=\frac{e^{-k_o}}{2^{n+1}}G_n(k_o)\text{(See Yim 1962a)}$$

$$G_{-1}\left(\frac{k_of_1}{2}\right)=\frac{k_of_1}{2}\left\{K_1\left(\frac{k_of_1}{2}\right)-K_o\left(\frac{k_of_1}{2}\right)\right\}$$

$$G_{-2}\ (k_o)=\frac{k_o}{3}\left\{K_1(k_o)+2\,k_oK_o(k_o)-2k_o\,K_1\ (k_o)\right\}$$

$K_o(k_o)$, $K_1(k_o)$ are the modified Bessel functions of the second kind of zeroth and first order respectively. Hence the value of ϵ_o which makes $R(I_o)$ minimum is $\epsilon_o = \frac{-\beta}{2\alpha}$

where

$$\alpha = \frac{2}{3} + 2e^{-k_o} G_{-2}(k_o) - 4 e^{-k_o/2} G_{-2}\left(\frac{k_o}{2}\right)$$

$$-2k_o \left\{ e^{-k_o f_1/2} G_{-1} \left(\frac{k_o f_1}{2}\right) - e^{-(k_o+k_o f_1)/2} G_{-1}\left(\frac{k_o+k_o f_1}{2}\right)\right\} + k_o^2 \frac{e^{-k_o f_1}}{2} K_o(k_o f_1)$$

$$\beta = -2k_o \left\{ e^{-k_o f_1/2} G_{-1} \left(\frac{k_o f_1}{2}\right) - e^{-(k_o+k_o f_1)/2} G_{-1} \left(\frac{k_o+k_o f_1}{2}\right)\right\}$$

$$+ k_o^2 e^{-k_o f_1} K_o (k_o f_1).$$

The values of ϵ_o for different Froude numbers and $f_1 = 1$ are given in Table 1. The values of ϵ_n for $n \geq 1$ can be calculated as above. However, since these are not so important in the usual range of Froude number, a rough and easily obtained approximation may be sufficient.

Using the method of stationary phase for $y = 0$

$$I_n = -8 \sqrt{\frac{\pi}{2k_1}} \left\{ \frac{\left(1-e^{-k_o}\right)}{k_o^n} \epsilon_n - (1+\epsilon_n)\frac{e^{-k_o f_1}}{k_o^n} \sum_{r=0}^{n} \frac{\left[f_1^{r+1} - (f_1-1)^{r+1} k_o^{r+1}\right]}{(r+1)!} \right\}$$

$$\text{X} \sin\left(k_1 x + \frac{\pi}{4}\right). \qquad [57,b]$$

If we put $I_n = 0$

$$\epsilon_n = \frac{\sum_{m=1}^{n+1} \left[\frac{(f_1 k_o)^m - \left\{ k_o(f_1 - 1) \right\}^m}{m!} \right]}{e^{k_o f_1} \left(1 - e^{-k_o} \right) - \sum_{m=1}^{n+1} \left[\frac{(f_1 k_o)^m - \left\{ k_o(f_1 - 1) \right\}^m}{m!} \right]} \qquad [57,c]$$

Equation [57,c] is evaluated for $f_1 = 1$ and different Froude numbers, from $n = 1$ to $n = 3$ are shown in Table 1. For $f_1 = 1$,

$$\epsilon_n = \frac{\sum_{m=1}^{n+1} \frac{k_o^m}{m!}}{e^{k_o} - 1 - \sum_{m=1}^{n+1} \frac{k_o^m}{m!}} \qquad [57,d]$$

We note that ϵ_n in [57,c] is not a function of a_n or b_n but only of f_1, k_o, and n, and is always positive for $f_1 \geq 1$ since the numerator in Equation [57,c] is positive, and the denominator in Equation [57,c] is larger than that of Equation [57,d]. The latter is positive due to the fact that

$$1 + \sum_{m=1}^{n+1} \frac{k_o^m}{m!}$$

is a partial sum of the series expansion of e^{k_o}. (This is also true for $y \neq 0$ in Equation [57,a]. Hence if the b_n's are all positive (as for the sine ship treated in the next Section), each horizontal section of the bulb represented by Equations [21] and [23,b] in $0 \leq z_1 \leq 1$ will be smaller than any corrected bulb.

Since we determined ϵ_n from the stationary phase at $y = 0$, ϵ_n's $(n \geq 1)$ in Table 1 are overestimated, or rather Equations [21] and [23,b] with $b_n (1+\epsilon_n)$ instead of b_n would indicate the upper bound of the size of bulb for minimum wave resistance subject to the conditions of this method.

In order to find the exact optimum doublet distribution on $0 \leq z_1 \leq f$ for the minimization of the wave resistance due to a given ship it is better to attack this problem by a somewhat different approach. That is, we determine the unknown coefficients b_n of the doublet distribution directly by minimizing the wave resistance of a given ship with the bulb (see Yim, 1962b). This is equivalent to the present method provided we obtain ϵ_n by minimizing the wave resistance due to the complete uncancelled wave system directly. ϵ_n is not necessarily always positive, since in this case there are many extra cross product terms.

According to Figures 1a and 2a, the exact optimum bulb is thinner near the free surface and thicker near the keel than the one corrected by the method given in this paper.

It should be mentioned here that there is no interaction between the bow and the stern waves for the ideal doublet distribution since in this case the bow and stern waves are separately cancelled out by the bow and stern bulbs respectively. However for the finite doublet lines there exist interactions and the curves of wave resistance versus Froude number have humps and hollows. Therefore the design Froude number of the ship would normally be selected so that it falls at a hollow on the resistance curve. However, since the optimum bulb at the bow or the stern for each Froude number has the effect of smoothing out the resistance curve to a great extent (Yim, 1962a) the unfavorable effect of not falling on a hollow is usually outweighted by the advantage of using an optimum bulb.

CASE OF SINE SHIP

Inui (1960) pointed out that the smaller the Froude number, the greater becomes the importance of the first term in Equation [16]. In the practical case we need not take many terms in the series. At moderate ship speeds even one term will be enough since the second term of the series [16] produces wave heights of the order $1/k_1^{\,2} < 0.01$ for $F = .3$ while the first term is considered to be of order 1 (see Appendix). This is natural since it is well known that the first term of the series [16] for the source distribution of the ship is proportional to the angle of entrance of a ship and the angle of entrance has a great influence on the wave resistance.

As an example, the first four terms of the cosine series will be taken for the source distribution in Equation [16]. Then the corresponding ship will be a sine ship in the Michell's sense. The coefficients a_n are taken such that the ship half-breadths are nondimensionalized with respect to the half beam and the stations are measured from the bow and nondimensionalized with respect to the ship length L. The values of a_n, b_n from Equation [21], and ϵ_n from Equation [57,c] for different Froude numbers and length-draft ratios are shown in Table 1. The corresponding doublet distributions, Equations [21] and [23,b] without perturbation corrections, in $0 \leq z_1 \leq 1$, for different Froude numbers are plotted as the solid curves in Figures 1a and 2a. Also shown in these figures for purpose of comparison, are corresponding curves determined from the exact ship wave resistance minimization theory based on finding directly the optimum doublet distribution for a given ship with a bulb which does not extend below the keel (see Yim, 1962b).

The two dimensional approximate relation of bulb radius r and the strength of doublet μ per unit length of z is $\mu = r^2V$. This relation, although overestimating the actual radius r, is used as an alternate means of expressing the nondimensional doublet strength in Figures 1a and 2a in order to give some idea of the approximate area distribution of the bulb.

The remarkable reductions of wave resistance due to the bulbs for both cases are shown in Figures 1b and 2b. The bulb sizes near the keel ($z = -1$) determined from Equation [21] are very much smaller than those shown by the exact method illustrated by the broken curves although near the free surface the former are slightly larger than the latter. The Froude number of least resistance, where the reduction of wave resistance is almost 90 percent, is usually less than the Froude number for which the calculation of the doublet distribution [21] was made. This is due to the fact that we have cut off the infinite ideal doublet distribution at the keel. The figures of doublet distributions [21], uncorrected by perturbations, appear to be almost linear in $0 \geq z \geq -1$. This indicates the importance of the first term $b_o a_1$ of the series for the doublet distribution [21] or the first term a_o of the series for the source distribution for a ship in the practical range of Froude numbers, as pointed out earlier and in the Appendix. That is, the equation for the first term in the doublet series $b_o = a_o/k_1$ is a very simple but very important approximate relation between the ship and the corresponding bulb. It is clear from the equation for the perturbation [57], and Table 1 that the nonlinearity increases when the length of bulb becomes smaller. Hence it may be desirable to increase the magnitude of the doublet strength near the keel or rather extend the doublet distribution below the keel if possible according to the methods indicated in Equation [57]. However, according to Figures 1b and 2b the doublet distribution of Equation [21] which gives a much smaller bulb than the exact optimum one results in a very favorable effect even without the perturbation corrections. Besides even these bulbs are quite large when the Froude number is large. In fact we may expect that the effects of viscosity, cavitation, and other design conditions would ultimately determine the maximum bulb size at these Froude numbers.

CASE OF THE PARABOLIC SHIP

When we consider the source distribution

$$m = c\ (1 - 2x) \quad \text{in } 0 \leq z_1 \leq 1 \qquad [58]$$

Michell's linear ship relation

$$m(x) = \frac{1}{2\pi} \frac{\partial y}{\partial x}$$

will indicate that this is a parabolic ship with

$$c = \frac{1}{\pi} \frac{B}{L}$$

where B/L is the beam-length ratio.

Since we have an even power and an odd power term in Equation [58], the bow waves contain both the sine and cosine components. To cancel the sine bow waves we need to put a doublet line of the strength from Equation [23,b]

$$\frac{\mu}{c} = \frac{z_1}{k_1} \qquad 0 \leq z_1 \leq 1$$

$$\frac{\mu}{c} = \frac{1}{k_1} \qquad 1 \leq z_1$$

The effect of doublet lines, both the ideal distribution cut off at the keel and the optimum finite distribution has already been demonstrated for a sine ship. In Figures 3a and 3b the optimum finite doublet distribution for the parabolic ship and the corresponding bow wave resistance with and without the doublet distribution are shown (the process of calculation is discussed in Yims paper of 1962b).

To cancel the cosine bow waves we need to superimpose the quadrupole line of the strength from Equations [28] and [29],

$$\frac{\lambda}{c} = \frac{k_0}{k_1^{\,3}} \quad z_1^{\,2} \qquad\qquad \text{in } 0 \le z_1 \le 1$$

$$\frac{\lambda}{c} = \frac{k_0}{k_1^{\,3}} [z_1^{\,2} - (z_1-1)^2] = k_0 \frac{(2z_1-1)}{k_1^{\,3}} \quad \text{in } 1 \le z_1$$

k_1 is usually larger than 1 and quite large if Froude number is small. k_0 is smaller by the factor of the draft length ratio than k_1. Hence inequality (45) would be satisfied for quite a large z_1 for moderate Froude numbers but eventually inequality (45) is violated for z_1 larger than a certain number. However the effect of cutting the line off at a large z_1 may not be serious as was shown for the case of doublet.

The wave resistance due to the cosine bow waves of the parabolic ship with and without the ideal quadrupole line cut off at the keel are shown in Figure 4b. The optimum quadrupole strength and the wave resistance with and without the optimum quadrupole (see Yim 1962b) is also shown in Figures 4a and 4b. Although the effect of the quadrupole line is not as prominent as that of the doublet for the sine wave, even the ideal quadrupole line cut off at keel is shown to be quite favorable for moderate Froude numbers. The wave resistance due to the cosine component of the bow waves even without the bulb as is shown in Figure 4b, is quite small compared to that due to the sine component. However, in practice the cosine components seem to be much larger than in this theory due to the change of the actual body streamlines caused by boundary layers and wakes.

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APPENDIX

Since a_{2n} is the coefficient of a power series which represents the source distribution of a ship, it is usually comparable to the cosine series where $a_{2n} = (-1)^n \frac{\pi^{2n}}{(2n)!}$. It therefore seems reasonable to assume that $(2n)!\, a_{2n} < M\pi^{2n}$, M = finite number, for all n. For usual ships $F^2 = \frac{1}{K_1} < \frac{1}{4}$. Besides the integral

$$C \equiv \int_0^{\pi/2} \cos^{2n-1} \theta d\theta = \frac{2.4.---.2(n-1)}{3.5.---.(2n-1)}$$

is decreasing with increasing n. Hence a typical term in the Equation [18] for the wave height due to the ship can be estimated as

$$\left| \int_0^{\pi/2} e^{-k_o f \sec^2\theta} \frac{(2n)!a_{2n}}{(k_1 \sec \theta)^{2(n-1)}} \sin(k_1 x \sec \theta)\cos(k_1 y \sin\theta\sec^2\theta)d\theta \right|$$

$$< e^{-k_o f} M\pi^2 \int_0^{\pi/2} \frac{\cos^{2(n-1)}\theta}{(k_1/\pi)^{2(n-1)}} d\theta = \frac{\pi^2 MC}{(k_1/\pi)^{2(n-1)}} e^{-k_o f} \quad [A1]$$

Since π/k_1 is a small number usually, it is clear how rapidly the integral [A1] decreases when n increases. Hence in practice we do not need to take many terms of the series in determining the optimum bulb. That is, the magnitude of the corresponding terms for the doublet distribution decrease very rapidly in the usual range of Froude number. Therefore we have only to consider the case of finite n when we consider the limiting case of $z_1 \to \infty$; i.e. the effect of the infinitely long doublet distribution from Equation [20,a]. For the case of a finite doublet distribution we have

$$\zeta_B = - 8 \int_0^{\pi/2} \sum_{n=0}^{\infty} b_n n! \left(\frac{e^{-k_o f \sec^2\theta}}{k_o^{n-1} \sec^2(n-1)\theta} - e^{-k_o f_1 \sec^2\theta} \sum_{r=C}^{n} \frac{(f_1 - f)^{n-r}}{(n-r)!\, k_o^{r-1} \sec^2(r-1)_\theta} \right) \sin(k_1 x \sec\theta)$$

$$X \cos(k_1 y \sin\theta \sec^2\theta)\, d\theta$$

Let us denote the terms due to the limit f_1 by R_n. Then for a finite n

$$\lim_{f_1 \to \infty} R_n \to 0$$

because

$$\lim_{f_1 \to \infty} e^{-k_o f} (f_1 - f)^n = 0$$

The argument is the same when the amplitude functions are given by Equation [22].

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TABLE 1

VALUES OF b_n, ϵ_n FOR AN APPROXIMATE SINE SHIP

L/H	F_H	b_o	b_1	b_2	b_3	ϵ_o	ϵ_1	ϵ_2	ϵ_3
16	1.00	0.18750	3.6143-03	4.6448-05	4.4768-07	0.65940	6.8718	32.290	171.71
	1.25	0.29296	8.8241-03	1.7719-04	2.6684-06	0.96717	16.346	111.19	895.72
	1.5	0.42187	1.8298-02	5.2907-04	1.1474-05	1.2008	33.095	313.10	3588.6
	1.75	0.57421	3.3899-02	1.3341-03	3.9379-05	1.3657	60.207	761.61	11815.
24	1.00	0.12500	1.0709-03	6.1166-06	2.6202-08				
	1.25	0.19531	2.6145-03	2.3333-05	1.5617-07				
	1.5	0.28125	5.4215-03	6.9672-05	6.7152-07				
	1.75	0.38281	1.0044-02	1.7569-04	2.3048-06				
	2.00	0.50000	1.7134-02	3.9147-04	6.7076-06				

($3.6143\text{-}03 = 3.6143.10^{-3}$)

(3 X COEFFICIENTS OF SINE SHIP: $a_o = 3.$, $a_2 = -14.804$, $a_4 = 12.176$, $a_6 = -4.0057$)

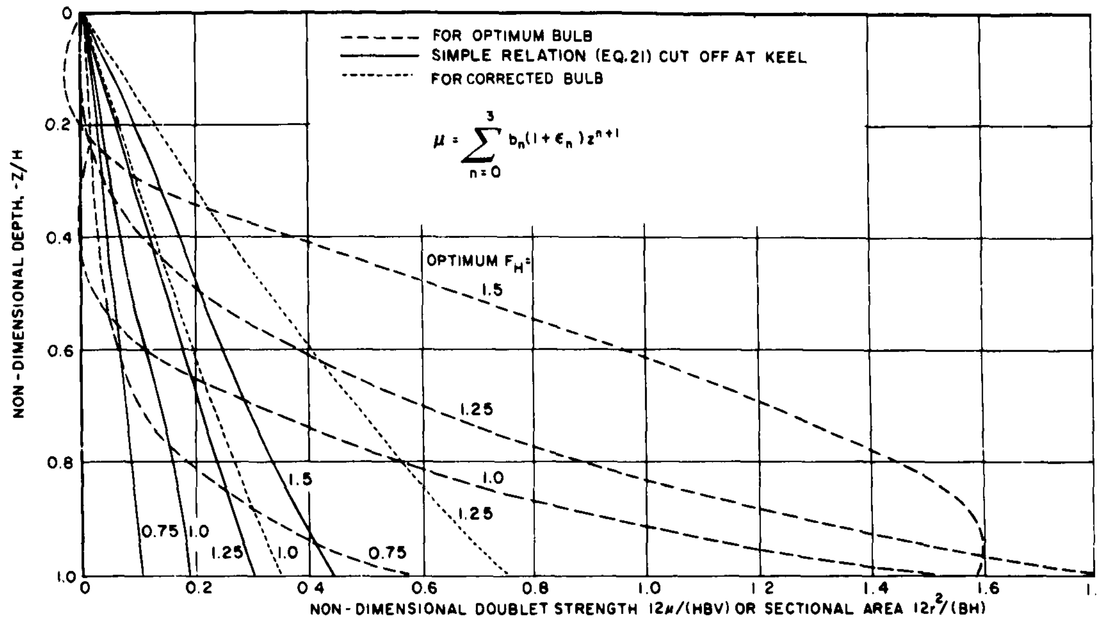


FIGURE 1a DOUBLET DISTRIBUTION FOR SINE SHIP (L/H=16)

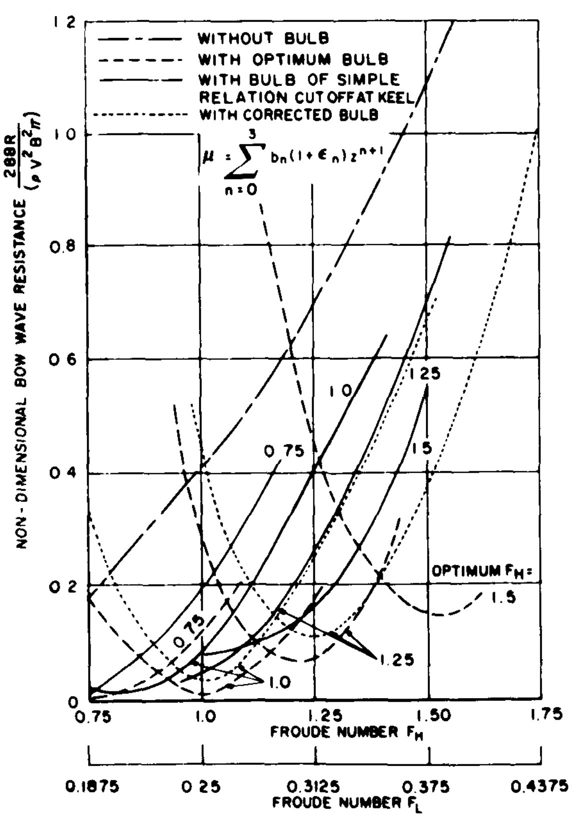


FIGURE 1b- BOW WAVE RESISTANCE OF SINE SHIP (L/H=16)

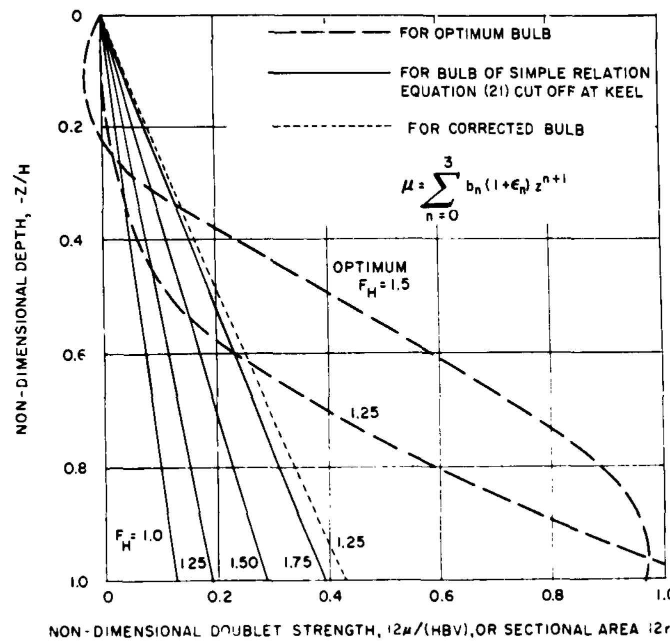


FIGURE 2a - DOUBLET DISTRIBUTION FOR SINE SHIP (L/H = 24)

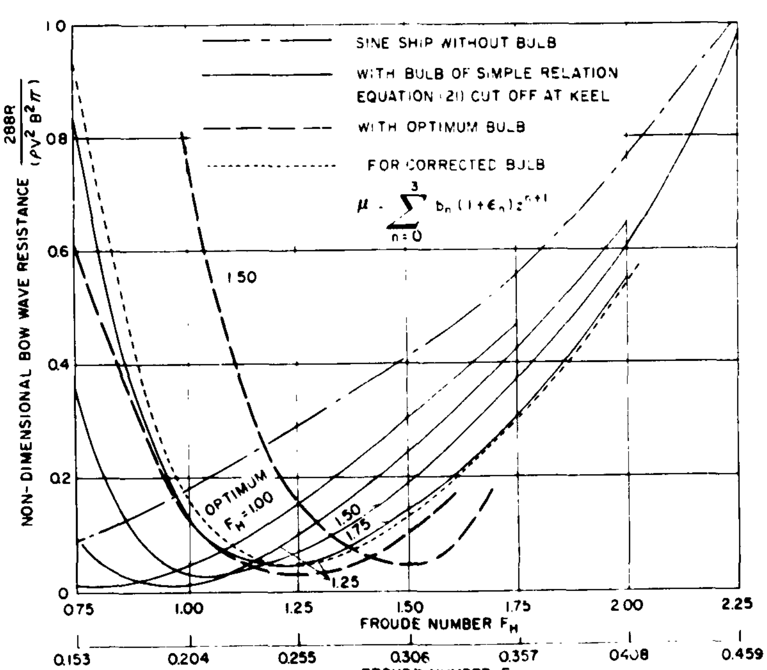
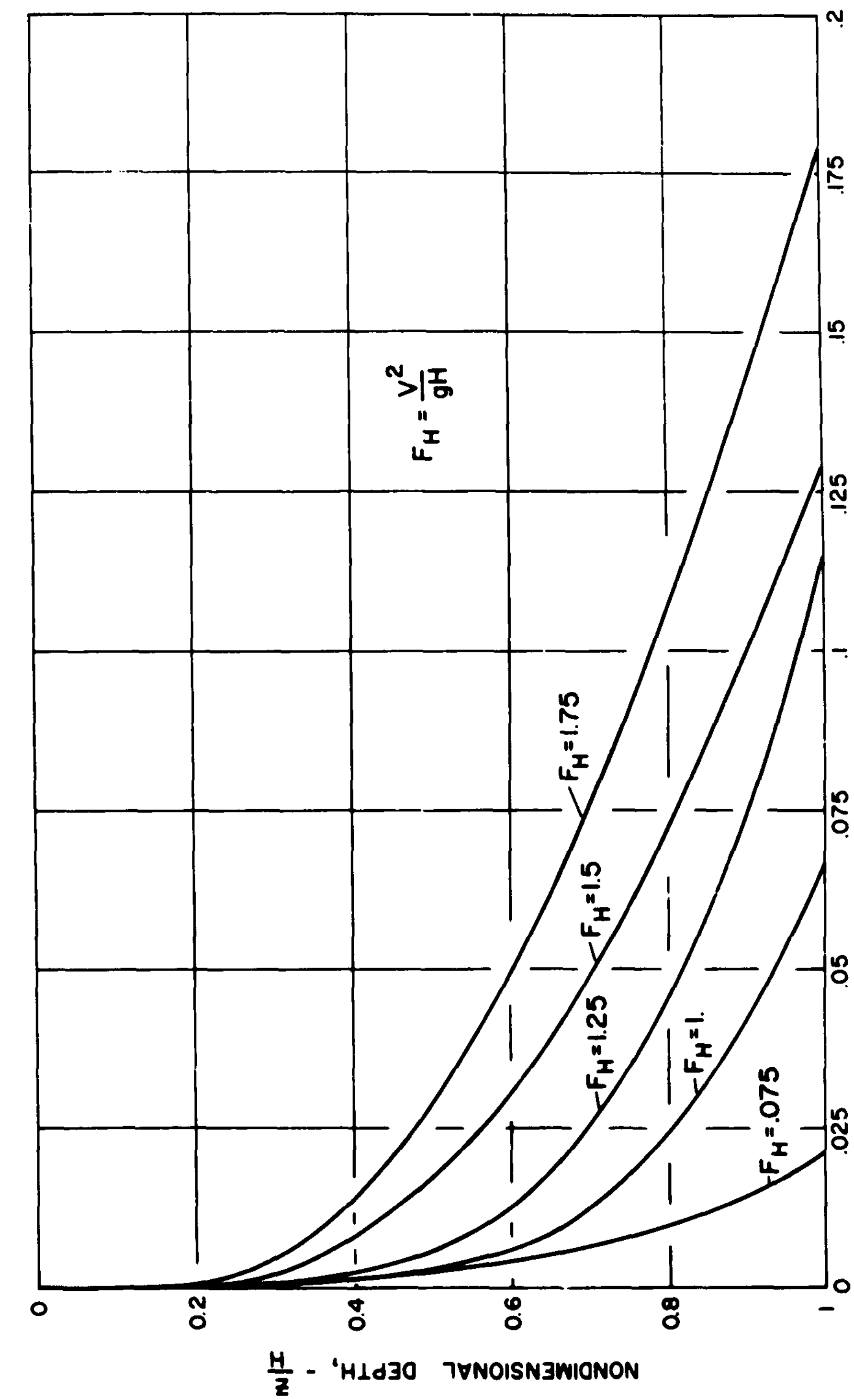


FIGURE 2b - BOW WAVE RESISTANCE OF SINE SHIP ($L/_H$ = 24)



NONDIMENSIONAL DOUBLET STRENGTH, $\frac{3\pi\mu}{(20\,VBH)}$

FIGURE 3a DOUBLET DISTRIBUTION FOR A PARABOLIC SHIP ($\frac{L}{H} = 16$)

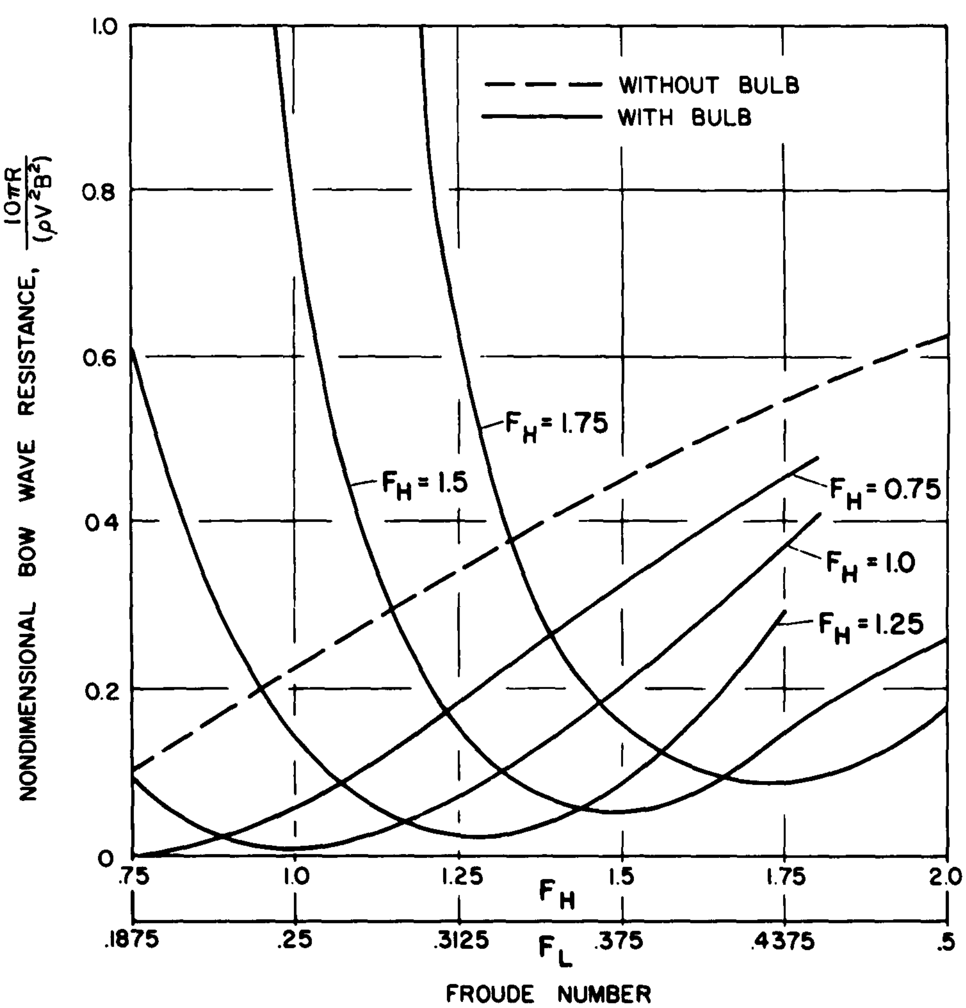


FIGURE 3b BOW WAVE RESISTANCE DUE TO THE EVEN TERM IN THE SOURCE DISTRIBUTION OF A PARABOLIC SHIP

$\left(\frac{L}{H} = 16\right)$

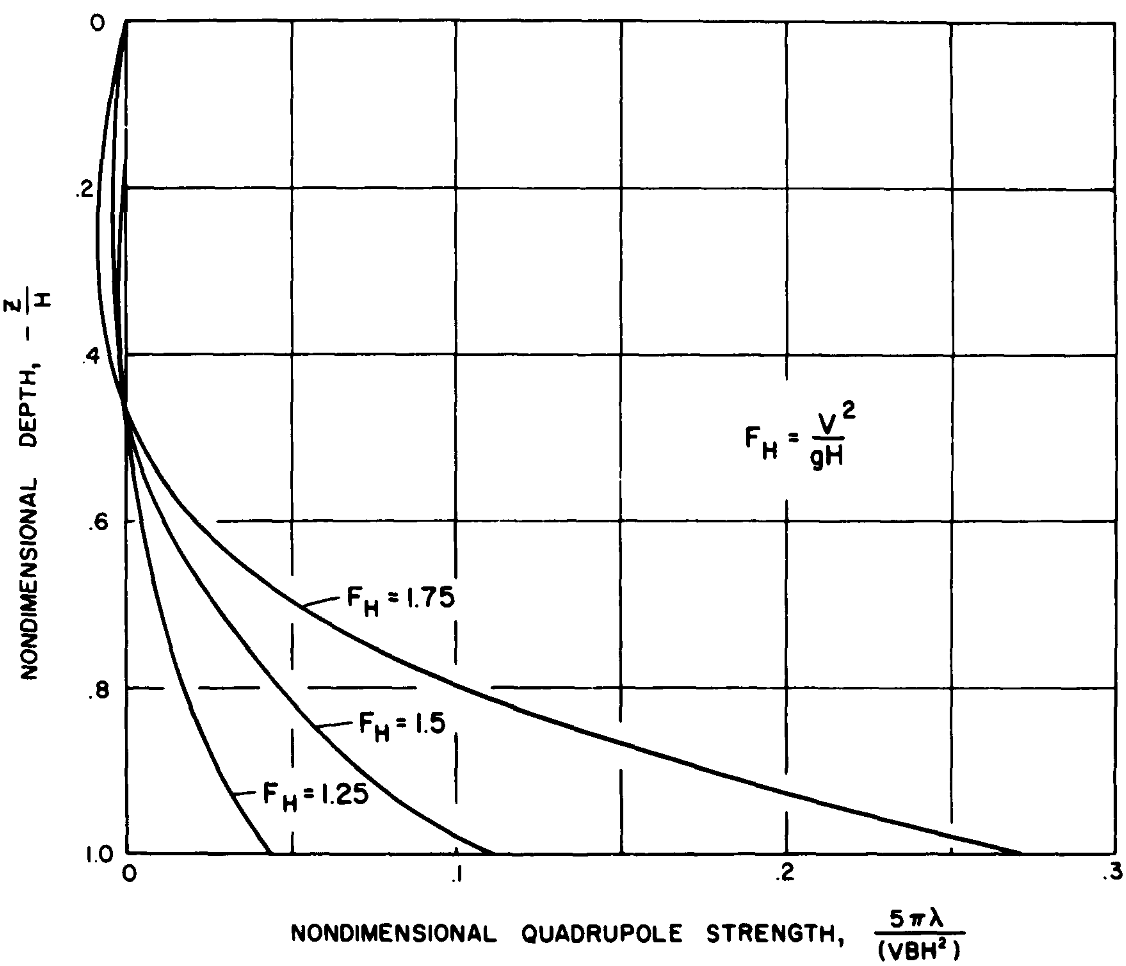


FIGURE 4a QUADRUPOLE DISTRIBUTION FOR A PARABOLIC SHIP, $(\frac{L}{H} = 16)$

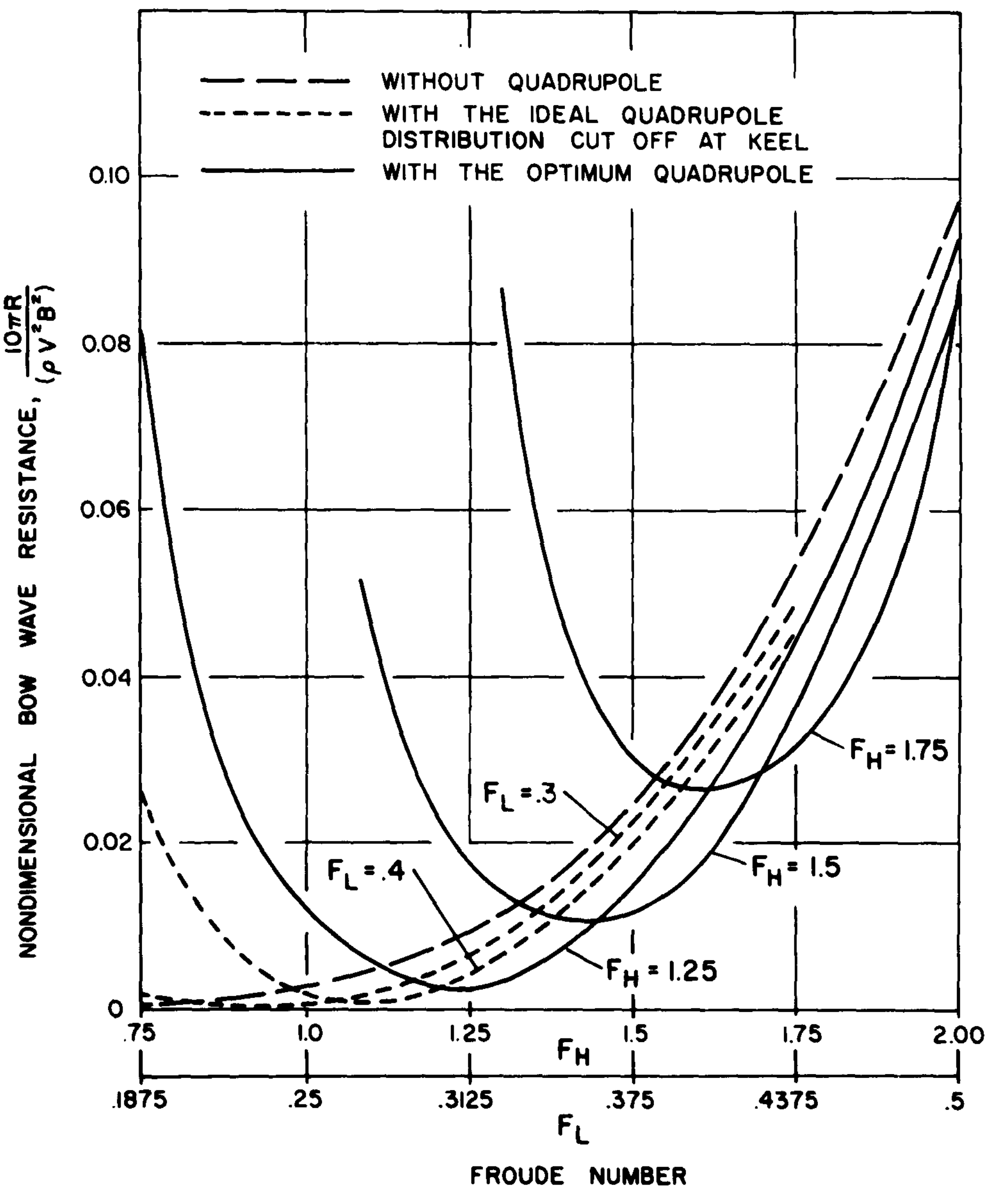
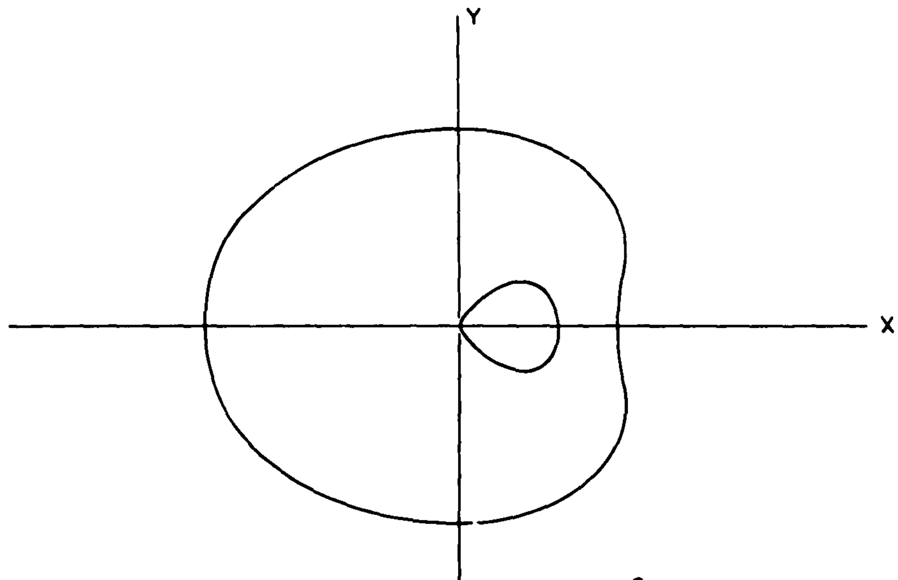
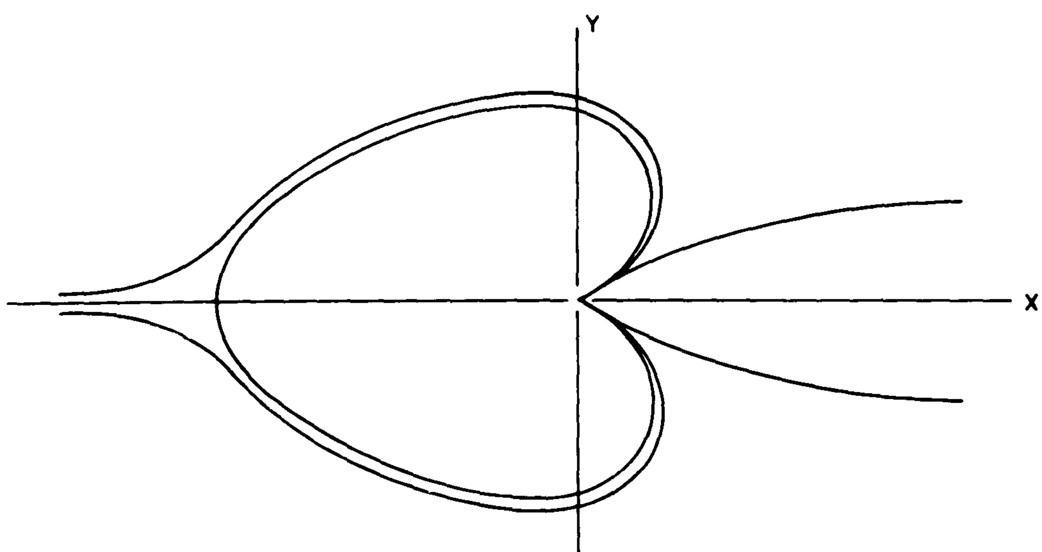


FIGURE 4b BOW WAVE RESISTANCE DUE TO THE ODD TERM IN THE SOURCE DISTRIBUTION OF A PARABOLIC SHIP, $(\frac{L}{H} = 16)$



(a) WHEN $\frac{\lambda}{V} \leq (\frac{\mu}{3V})^{\frac{3}{2}}$



(b) WHEN $\frac{\lambda}{V} > (\frac{\mu}{3V})^{\frac{3}{2}}$

FIGURE 5 SCHEMATIC DIAGRAM OF DIVIDING STREAMLINES DUE TO A POINT DOUBLET PLUS A POINT QUADRUPOLE AT THE ORIGIN WITH THE DIRECTIONS PARALLEL TO THE AXIS

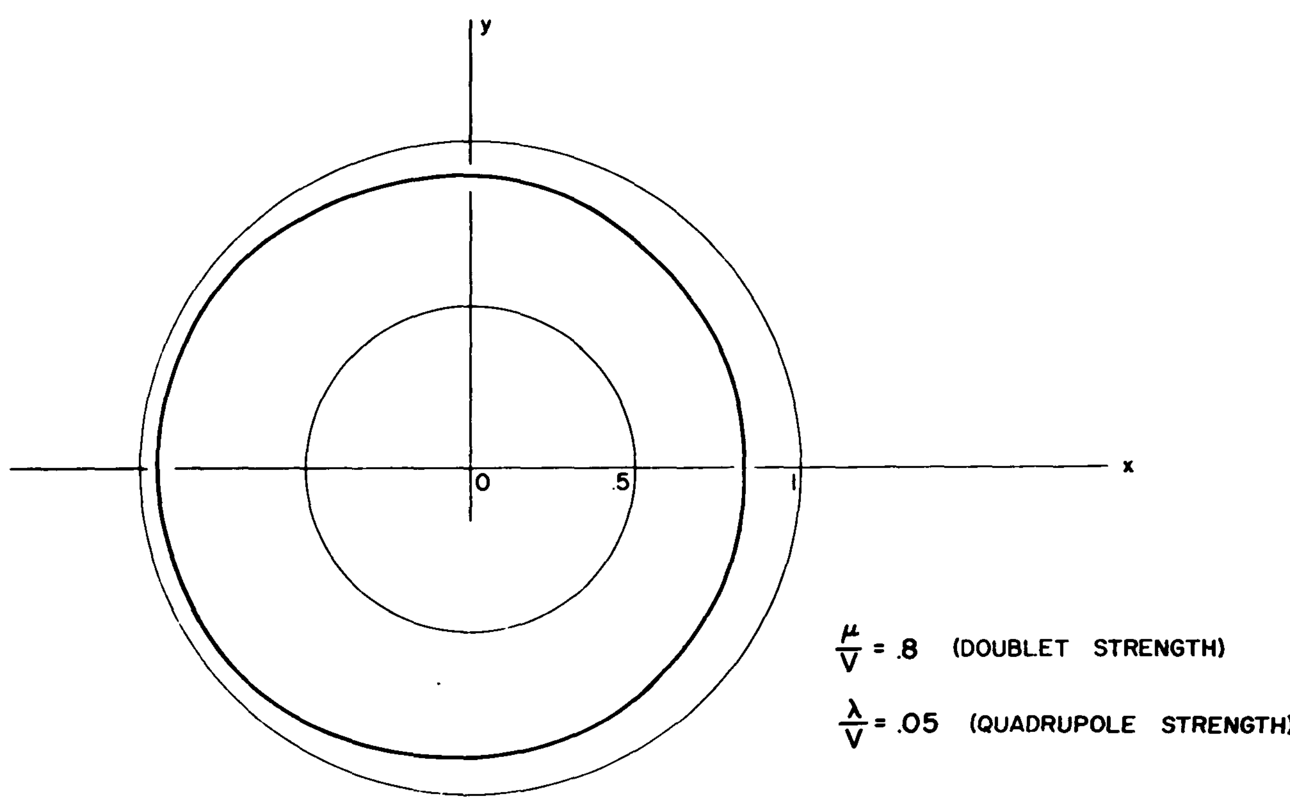


FIGURE 6 TWO DIMENSIONAL DIVIDING STREAMLINE DUE TO A POINT QUADRUPOLE AND A POINT DOUBLET LOCATED AT THE ORIGIN

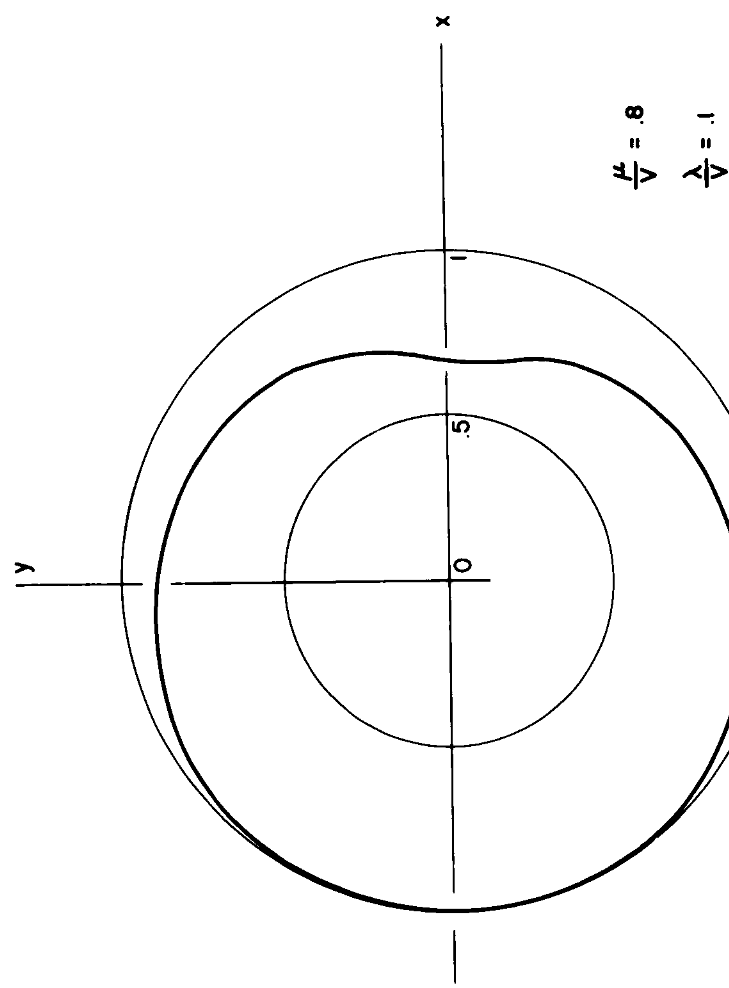
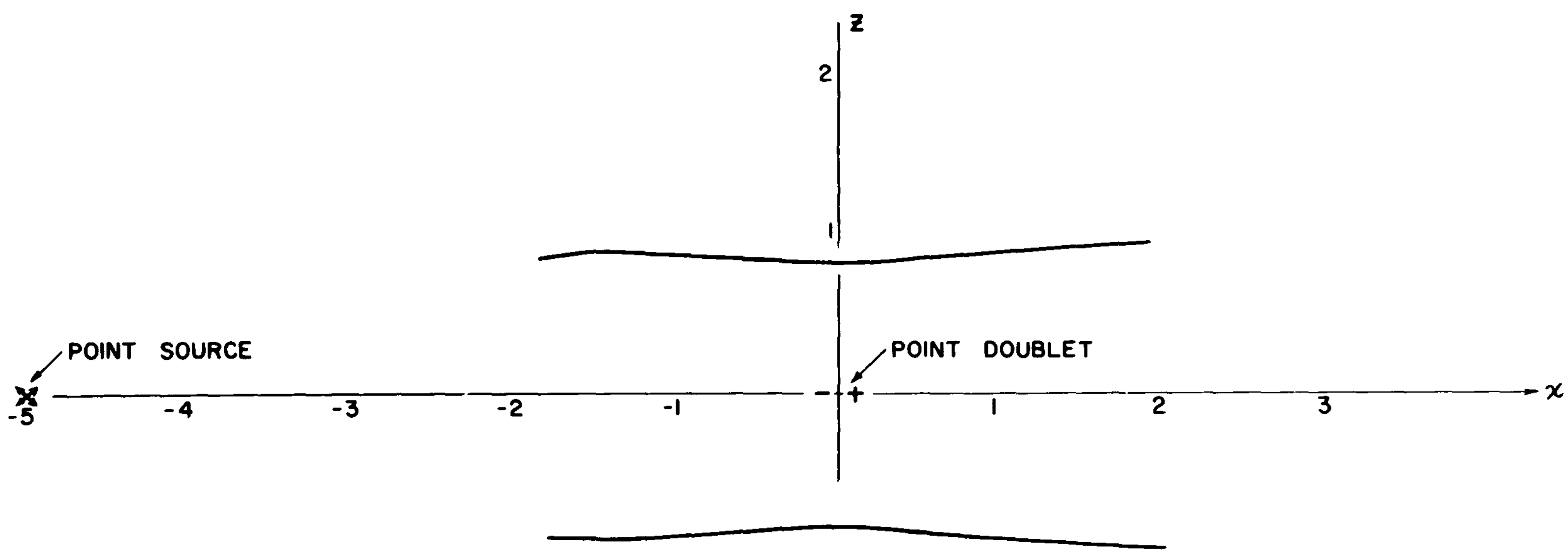
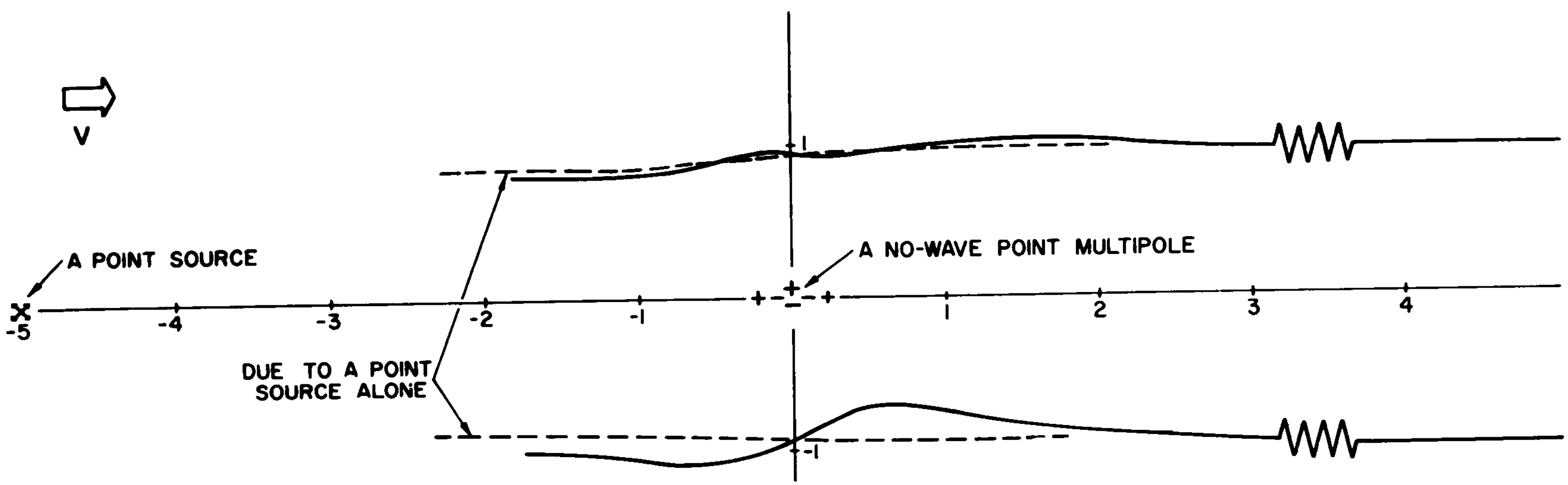


FIGURE 7 TWO DIMENSIONAL DIVIDING STREAMLINE DUE TO A POINT QUADRUPOLE AND A POINT DOUBLET LOCATED AT THE ORIGIN



$\bar{\mu} = .1$ IN Eqn (49)

FIGURE 8 TWO DIMENSIONAL DIVIDING STREAMLINE DUE TO A POINT SOURCE AND A POINT DOUBLET



$\bar{\mu} = 0.2$, $\bar{\lambda} = 0.1$ IN Eqn (53)

FIGURE 9 TWO DIMENSIONAL DIVIDING STREAMLINE DUE TO A POINT SOURCE AND A NO-WAVE POINT MULTIPOLE

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